Hamiltonian of the quantum and classical Ising model with skew magnetic field

This text is part of section 2 and Appendix B of Ref. [1].

Upon a suitable choice of the coordinates axes, the hamiltonian of the one-dimensional quantum Ising model with arbitrary normalized spin S and constant external magnetic field with arbitrary orientation is

$$\mathbf{H} = \sum_{i=1}^{N} \left(J s_i^z s_{i+1}^z - h_y s_i^y - h_z s_i^z \right), \tag{1}$$

where s_i^y and s_i^z stand for the y and z components, respectively, of the *arbitrary normalized spin operator*, defined as $\vec{s}_i \equiv \frac{\vec{S}_i}{\sqrt{S(S+1)}}$, $i \in \{1, 2 \cdots N\}$. The components of $\vec{S}_i$ are the spin S matrices, with norm $||\vec{S}||^2 = S(S+1)$, $S = 1/2, 1, 3/2, \cdots \infty$.

The chain has N spatial sites and satisfies periodic spatial boundary conditions. The coupling strength J between first-neighbor z-components of spin can either be positive (antiferromagnetic case) or negative (ferromagnetic case). Due to the rotational symmetry of the hamiltonian with respect to the z-axis (the easy-axis), the most general constant external magnetic field that we must consider is: $\mathbf{h} = h_y \hat{\jmath} + h_z \hat{k}$, where h_y and h_z are constants.

By taking the limit $S \to \infty$ in Eq. (1) we recover the classical version of the model; its corresponding thermodynamics is finite. When Eq.(1) is written in terms of the non-normalized operators S_i^y and S_i^z, the coupling constant becomes $J' = J/S(S+1)$ and the components of the magnetic field are $h'_y = h_y/\sqrt{S(S+1)}$ and $h'_z = h_z/\sqrt{S(S+1)}$[2, 3].

The classical limit of the Ising model with normalized arbitrary spin and skew constant magnetic field is obtained by replacing in hamiltonian (1) the components of normalized spin operators by their respective classical expressions

$$s_i^y = \sin(\theta_i) \quad \text{and} \quad s_i^z = \cos(\theta_i), \tag{2}$$

where θ_i is the angle between the normalized spin vector and the z axis (the easy-axis). Due to the rotational symmetry of the model with respect to this axis, $\theta_i \in [0, \pi/2]$ and $i = 1 \ldots N$. Substituting the relation (2) in hamiltonian (1), we obtain its classical version,

$$\mathcal{H}_{class} = \sum_{i=1}^{N} \left[J \cos(\theta_i) \cos(\theta_{i+1}) - h_y \sin(\theta_i) - h_z cos(\theta_i) \right], \tag{3}$$

where h_y and h_z are arbitrary constants.

References

[1] E.V. Corrêa Silva, James E.F. Skea, Onofre Rojas, S.M. de Souza and M.T. Thomaz,*Thermodynamics of the quantum and classical Ising model with skew magnetic field*, Physica **A387** (2008) 5117-5126 [http://dx.doi.org/10.1016/j.physa.2008.05.033].

[2] O. Rojas, S. M. de Souza and W.A. Moura-Melo, Phys. **A373**, 324 (2007).

[3] O. Rojas, S. M. de Souza, E. V. Corrêa Silva, and M. T. Thomaz, Eur. Phys. J. **B47**, 165 (2005).