

Hamiltonian of the quantum XXZ model with a single ion-anisotropy term

This text is part of section 2 of Ref.[1].

The Hamiltonian of the anisotropic spin- S XXZ with a single-ion anisotropy term and in the presence of an external magnetic field,

$$\mathbf{H} = \sum_{i=1}^N J' (\mathbf{S}_i, \mathbf{S}_{i+1})_{\Delta} - h' S_i^z + D' (S_i^z)^2. \quad (1)$$

We use the notation: $(\mathbf{S}_i, \mathbf{S}_{i+1})_{\Delta} \equiv S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z$. Here, S_i^x , S_i^y and S_i^z stand for the spin- S matrices in the i -th site of the chain and norm $\sqrt{S(S+1)}$; N is the number of sites in the periodic chain; J' is the exchange integral; Δ is the anisotropy constant in the z -direction; h' is the external magnetic field in the z -axis and D' is the single-ion anisotropy parameter.

We define a rescaled spin operator $\mathbf{s} \equiv \mathbf{S}/\sqrt{S(S+1)}$. This rescaled spin operator has unitary norm for all values of S . Rewriting the Hamiltonian (1) in terms of s and redefining the parameters $J \equiv S(S+1)J'$, $h \equiv \sqrt{S(S+1)}h'$ and $D \equiv S(S+1)D'$, we obtain

$$\mathbf{H} = \sum_{i=1}^N J (\mathbf{s}_i, \mathbf{s}_{i+1})_{\Delta} - h s_i^z + D (s_i^z)^2. \quad (2)$$

This also describes the dynamics of the XXZ model of a spin of unitary norm and $(2S+1)$ z -components. We continue to use the notation: $(\mathbf{s}_i, \mathbf{s}_{i+1})_{\Delta} \equiv s_i^x s_{i+1}^x + s_i^y s_{i+1}^y + \Delta s_i^z s_{i+1}^z$. Here, s_i^x , s_i^y and s_i^z stand for the spin- s ($s = 1/2, 1, 3/2, \dots$) rescaled matrices in the i -th site of the chain; N is the number of sites in the periodic chain; and Δ is the anisotropy constant in the z -direction.

The Helmholtz free energy of the classical version of the hamiltonian (2) is obtained by taking the limit $S \rightarrow \infty$ in the β -expansion of the Helmholtz free energy of the quantum hamiltonian (2).

References

- [1] Onofre Rojas, S.M. de Souza, E.V. Corra Silva and M.T. Thomaz, " *Thermodynamics of the quantum spin- S XXZ chain*", The European Physical Journal- Condensed Matter and Complex Systems B vol. 46 (2005) 385-398; Erratum: The European Physical Journal- Condensed Matter and Complex Systems B vol. 47 (2005) 165. [<http://dx.doi.org/10.1140/epjb/e2005-00310-5>]